GRADE 7

Every student should understand and use all concepts and skills from the previous grade levels. The standard is designed so that new learning builds on preceding skills. Communication, Problem-solving, Reasoning & Proof, Connections, and Representation are the process standards that are embedded throughout the teaching and learning of all mathematical strands.

Strand 1: Number and Operations

Number sense is the understanding of numbers and how they relate to each other and how they are used in specific context or real-world application. It includes an awareness of the different ways in which numbers are used, such as counting, measuring, labeling, and locating. It includes an awareness of the different types of numbers such as, whole numbers, integers, fractions, and decimals and the relationships between them and when each is most useful. Number sense includes an understanding of the size of numbers, so that students should be able to recognize that the volume of their room is closer to 1,000 than 10,000 cubic feet. Students develop a sense of what numbers are, i.e., to use numbers and number relationships to acquire basic facts, to solve a wide variety of real-world problems, and to estimate to determine the reasonableness of results.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, and the relationships among numbers and different number systems.

In Grade 7, students extend their work with equivalency among fractions, decimals and percents to order and compare them. They work with common factors and multiples as they deal with prime and composite numbers. Students are introduced to absolute value and build fluency with all rational numbers.

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
PO 1. Recognize and convert between expressions for positive and negative rational numbers, including fractions, decimals, percents, and ratios.	M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	In addition to general translations among these representations, students are expected to demonstrate quick recall of equivalent forms of common fractions, decimals, ratios, and percents. They are also expected to translate between negative fractions and decimals.
Connections: M07-S1C1-03, M07-S1C2-02, M07-S1C2-03, M07-S1C3-01, M07-S1C3-02, M07-S2C2-01, M07-S3C3-02		Continued on next page

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		
		Examples: • Represent 40% as a fraction and as a decimal. • Write $-\frac{13}{20}$ as a decimal.
PO 2. Find or use factors, multiples, or prime factorization within a set of numbers.		Various models and representations could include but are not limited to tree diagrams, lists of factors, or arrays.
Connections: M07-S1C3-01, M07-S1C3-03		
PO 3. Compare and order rational numbers using various models and representations. Connections: M07-S1C1-01, M07-S1C2-02, M07-S1C3-01	M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Various models and representations could include but are not limited to number line, coordinate graph, or physical models. Examples: • List the numbers $\frac{2}{3}$, $-\frac{2}{3}$, 1.2 , $\frac{4}{3}$, $-\frac{4}{3}$, -1.2 , and $-\frac{7}{4}$ in increasing order and then locate the numbers on the number line. • Compare each pair of numbers using <, >, or =. • $-\frac{11}{21} \Box -\frac{13}{21}$ • $-\frac{7}{5} \Box -1.35$ • $2\frac{3}{4} \Box 2.75$

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:	MO7 CEC2 04 Depresent a problem	Evennes
PO 4. Model and solve simple problems involving absolute value. Connections: M07-S1C2-02	M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution	 Examples: Explain why 5 and -5 have the same absolute value. What is the absolute value of -8? Demonstrate on a number line -8 .
	M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	-8 -7 -6 -5 -4 -3 -2 -1 0

Strand 1: Number and Operations Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

In Grade 7, students' understanding of equality is essential to number operations and algebraic reasoning as they further develop these important ideas. Students build on their previous work in adding, subtracting, multiplying, and dividing fractions and they use rational numbers in solving problems. They develop fluency and flexibility with efficient procedures including the standard algorithm for all operations with integers. Students understand why the procedures work and use them to solve problems. They continue to build on applying order of operations to numerical expressions.

Performance Objectives	Process Integration	Explanations and Examples
Performance Objectives Students are expected to: PO 1. Add, subtract, multiply, and divide integers. Connections: M07-S1C2-02, M07-S1C2-05, M07-S3C1-01, M07-S3C3-02, M07-S3C3-03, M07-S3C3-05, M07-S3C3-06	M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution M07-S5C2-05. Apply a previously used	Explanations and Examples Representing operations in multiple ways is critical. This is an opportunity for students to demonstrate the connections among the operations and to show what is the same and different in performing these operations with different types of numbers. Visual representations may be helpful as students begin this work; they become less necessary as students become more fluent with the operations.
	problem-solving strategy in a new context.	• Use a picture or physical objects to illustrate: o 3 - 7 o -3 - 7 o -3 - (-7) o (-3)(-7) o 21 ÷ (-3) • At noon on a certain day, the temperature was 13°; at 10 p.m. the same day, the temperature was -8°. How many degrees did the temperature drop between noon and 10 p.m.?

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
PO 2. Solve problems with rational numbers and appropriate operations using exact answers or estimates. Connections: M07-S1C1-01, M07-S1C1-03, M07-S1C1-04, M07-S1C2-01 M07-S1C3-02	M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	 Determine a reasonable estimate for \$\frac{3}{13}\$ x \$\frac{20}{41}\$, 0.28 \div 0.96, and -0.23 x 12.4. Tom wants to buy some candy bars and magazines for a trip. He has decided to buy three times as many candy bars as magazines. Each candy bar costs \$0.70 and each magazine costs \$2.50. The sales tax rate on both types of items is \$6\frac{1}{2}\%\$. How many of each item can he buy if he has \$20.00 to spend?
PO 3. Solve problems involving percentages, ratio and proportion, including tax, discount, tips, and part/whole relationships. Connections: M07-S1C1-01, M07-S1C3-02	M07-S5C2-01. Analyze a problem situation to determine the question(s) to be answered. M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language. M07-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Students should be able to explain or show their work using a representation (numbers, words, pictures, physical objects, or equations) and verify that their answer is reasonable. Examples: • At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs and is going to give all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify your solution. • After eating at a restaurant, you know that the bill before tax is \$52.60 and that the sale tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount. How much should you leave for the waiter? How much will the total bill be, including tax and tip? Explain the steps to determine your solution.

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
PO 4. Represent and interpret numbers using scientific notation (positive exponents only).	M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	 Examples: Represent 4.27 x 10³ in standard form. Represent 18,300,000 in scientific notation.
PO 5. Simplify numerical expressions using the order of operations and appropriate mathematical properties. Connections: MOZ S1C3 01, MOZ S2C3		Mathematical properties for this grade level include commutative, distributive, associative, identity, and inverse properties. Expressions should include no more than 5 operations.
Connections: M07-S1C2-01, M07-S3C3-02		Example: • Simplify the expression. Explain each of your steps in the process. $(-3)^2 + (4-7) \div 3$

Strand 1: Number and Operations

Concept 3: Estimation

Use estimation strategies reasonably and fluently while integrating content from each of the other strands.

In Grade 7, students use estimation skills to verify their work. They continue to make reasonable estimates using integers, fractions, decimals, and percents in problems involving all operations. Students connect their work with estimation to their work with measurement when estimating measured quantities and converting between measures.

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to: PO 1. Estimate and apply benchmarks for rational numbers and common irrational numbers. Connections: M07-S1C1-01, M07-S1C1-02, M07-S1C1-03, M07-S1C3-03, M07-S2C2-04, M07-S4C4-01, M07-S5C2-11	M07-S5C2-02. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem. M07-S5C2-05. Apply a previously used problem-solving strategy in a new context.	Examples: • Use 3 as an approximation for pi. • $\frac{7}{8} + \frac{7}{8}$ is approximately 2 because $\frac{7}{8}$ is a value close to 1
PO 2. Make estimates appropriate to a given situation. Connections: M07-S1C1-01, M07-S1C2-02, M07-S1C2-03, M07-S1C3-04, M07-S2C1-02, M07-S3C3-03, M07-S3C3-05, M07-S3C4-01, M07-S4C4-01, M07-S4C4-03, M07-S4C4-04, M07-S4C4-05, M07-S4C4-06	M07-S5C2-01. Analyze a problem situation to determine the question(s) to be answered. M07-S5C2-02. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem. M07-S5C2-03. Identify relevant, missing, and extraneous information related to the solution to a problem.	Students estimate using all four operations with whole numbers, fractions, and decimals. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimations are compared to actual calculations when appropriate. Estimation strategies for calculations with fractions and decimals extend from students' work with whole number operations.
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Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
		 Estimation strategies include, but are not limited to: front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts), clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate), rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values), using friendly or compatible numbers such as factors (students seek to fit numbers together - i.e., rounding to factors and grouping numbers together that have round sums like 100 or 1000), and using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate). Specific strategies also exist for estimating measures. Students develop fluency in estimating using standard referents (meters, yard, etc) or created referents (the window would fit about 12 times across the wall).
PO 3. Estimate square roots of numbers less than 1000 by locating them between		Example:Between which two consecutive integers does the
two consecutive whole numbers.		square root of 74 lie?
Connections: M07-S1C1-02, M07-S1C3 01	-	

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Performance Objectives Students are expected to: PO 4. Estimate the measure of an object in one system of units given the measure of that object in another system and the approximate conversion factor. Connections: M07-S1C3-02	M07-S5C2-01. Analyze a problem situation to determine the question(s) to be answered.	Students are expected to estimate within and between the US Customary and metric systems of measurement with conversion factors given. Conversions should include length, volume, temperature, mass, and temperature. Conversions should focus on commonly used values and contexts which are meaningful. Examples:
		 If 1 km = approx 8 of a mile, how many miles is a 5 km race. A gas tank for a car typically holds about 12 gallons of gas. How many liters does it hold if there are approximately 3.8 liters in a gallon? Joe was planning a business trip to Canada so he went to the bank to exchange \$200 US Dollars for Canadian dollars at a rate of \$1.15 CDN per \$1 US. On the way home from the bank, Joe's boss called to say that the destination of the trip had changed to Mexico City. Joe went back to the bank to exchange his Canadian dollars for Mexican pesos at a rate of 10.4 pesos per \$1 CDN. How many Mexican pesos did Joe get? [Note: currency conversion rates vary from day to day.]

Strand 2: Data Analysis, Probability, and Discrete Mathematics

This strand requires students to use data collection, data analysis, statistics, probability, systematic listing and counting, and the study of graphs. This prepares students for the study of discrete functions as well as to make valid inferences, decisions, and arguments. Discrete mathematics is a branch of mathematics that is widely used in business and industry. Combinatorics is the mathematics of systematic counting. Vertex-edge graphs are used to model and solve problems involving paths, networks, and relationships among a finite number of objects.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization, and representation to analyze and sort data.

In Grade 7, students apply their understanding of integers, fractions, decimals, and percents as they construct, analyze, and describe data in more complex situations that they may encounter in other school subjects and their lives.

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
PO 1. Solve problems by selecting, constructing, and interpreting displays of data including multi-line graphs and scatterplots.	M07-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Students will solve problems with multi-line graphs and scatterplots as well as all other representations taught in previous grade levels.
Connections: M07-S2C1-02, SC07-S1C3-01, SC07-S1C3-02, SC07-S1C3-03, SC07-S1C3-05, SC07-S1C4-01, SC07-S1C4-02, SS07-S1C1-01, SS07-S1C1-02, SS07-S1C1-03, SS07-S2C1-01, SS07-S2C1-02, SS07-S2C1-03, SS07-S4C1-01, SS07-S4C1-01, SS07-S4C1-02, SS07-S4C1-03		

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
PO 2. Interpret trends in a data set, estimate values for missing data, and predict values for points beyond the range of the data set. Connections: M07-S1C3-02, M07-S2C1-01, M07-S2C2-03, SC07-S1C3-01, SC07-S1C3-02, SC07-S1C3-03, SC07-S1C3-05	M07-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning. M07-S5C2-08. Make and test conjectures based on information collected from explorations and experiments.	Students will interpret trends using the graphical representations listed in M07-S2C1-01 and all other applicable representations taught in previous grade levels. In predicting values, students use approximate lines of best fit, interpolation, and extrapolation. Students identify correlations (i.e., positive, negative, and no correlation).
PO 3. Identify outliers and determine their effect on mean, median, mode, and range.	M07-S5C2-03. Identify relevant, missing, and extraneous information related to the solution to a problem. M07-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Outliers can be found in any data set or graphical representation. An outlier is numerical data that is significantly larger or smaller than the rest of the data in a set. At this grade level, students are expected to determine outliers by examining and comparing data but should be introduced to the idea that an outlier can be determined by mathematical calculations.
PO 4. Distinguish between a simple random and non-random sample.	M07-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	The school food service wants to increase the number of students who eat hot lunch in the cafeteria. The student council has been asked to conduct a survey of the student body to determine the students' preferences for hot lunch. They have determined two ways to do the survey. The two methods are listed below. Identify the type of sampling used in each survey option. Which survey option should the student council use and why? Write all of the students' names on cards and pull them out in a draw to determine who will complete the survey. Survey the first 20 students that enter the lunch room.

Strand 2: Data Analysis, Probability, and Discrete Mathematics Concept 2: Probability

Understand and apply the basic concepts of probability.

In Grade 7, students extend their knowledge of fractions by stating the theoretical probability of an event as a fraction, decimal, or percent based on the possible outcomes. They predict, record, and compare results in actual experiments. Experience with probability at this level will prepare students for deeper exploration of probability in the higher grades.

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to: PO 1. Determine conditional probabilities (experimental) in compound probability experiments. M07-S50 conjectur collected	Process Integration M07-S5C2-08. Make and test conjectures based on information collected from explorations and experiments.	Conditional probability is limited to situations with and without replacement. Students need multiple opportunities to perform compound experiments, record data, and calculate experimental probabilities. This builds upon prior knowledge from sixth grade in which students determine all of the possible outcomes in a sample space (M06-S2C2-03). Example: • Students conduct a bag pull experiment. A bag contains 12 marbles. There are six red marbles and six blue marbles. They draw two marbles at a time and record their results, before returning the marbles to the bag. The results from 50 trials were collected
		and are listed in the table below. Determine the experimental probability of drawing two red marbles, two blue marbles, or one of each kind of marble.
		Outcomes Frequency
		One of each 24 Two Red 14
		Two Red 14

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
PO 2. Experiment with two different events to determine whether the two events are dependent or independent of each other. Connections: M07-S2C2-01	M07-S5C2-08. Make and test conjectures based on information collected from explorations and experiments.	Determine whether the events are independent or dependent. Explain your choice. Selecting a marble and then choosing a second marble without replacing the first marble. Rolling a number cube and spinning a spinner.
PO 3. Compare the results of multiple repetitions of the same probability experiment to the theoretical probability. Connections: M07-S2C1-02	M07-S5C2-08. Make and test conjectures based on information collected from explorations and experiments.	Students should have opportunities to explore multiple repetitions of the same probability experiment (either by conducting the experiment themselves, compiling class data, or running simulations) so they can begin to understand that as the number of trials in an experiment increases, the more closely the experimental probability reflects the theoretical probability.
PO 4. Compare probabilities to determine fairness in experimental situations.		
Connections: M07-S1C3-01		

Strand 2: Data Analysis, Probability, and Discrete Mathematics Concept 3: Systematic Listing and Counting

Understand and demonstrate the systematic listing and counting of possible outcomes.

In Grade 7, students utilize graphic organizers to categorize data that may or may not include algebraic components. Students expand on their experience with counting problems by solving problems with increased rigor. Students continue to make connections to the multiplication principle of counting throughout the process of problem solving.

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
PO 1. Analyze relationships among the tree diagrams where items repeat and do not repeat; make numerical connections to the multiplication principle of counting.	M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	Students create and compare tree diagrams where objects repeat and do not repeat so as to analyze the numerical relationships in the number of ways these objects can be arranged. They come to recognize that a valid method of counting is to "over count" (by using the multiplication principle of counting) and divide out by the number of ways to arrange the repeating letters (since we only want to count these arrangements once).
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Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
	Example:	Example:
	F E R D R E R	F E E R E R E R E R E R E R E R E R E R
	R E F D F E F Start	Start R F E E E E E E E E E E E E E E E E E E
	F R D D R F R P R P R P R P R P R P R P R P R P	F R E E R R F E
	R F E R F R F R F R F R F R F R F R F R	E F R
	4 x 3 x 2 x 1	(4 x 3 x 2 x 1) (2 x 1)
		Continued on next page

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		Evample:
PO 2. Solve counting problems using Venn diagrams and represent the answer algebraically. Connections: M07-S3C3-01, M07-S5C2-09	M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	How many different ways can you arrange the letters in the word READ? How many different ways can you arrange the letters in the word REED? In the word, DEED? In the word, EPEE? Demonstrate the number of arrangements for each of these words using a tree diagram. Why are the number of arrangements for the words different? Which word has the most arrangements? Why? How might you figure out the number of arrangements mathematically without having to complete the entire tree diagram? Students may use a Venn diagrams, set notation or algebraic notation to solve counting problems. Students can be introduced to simple set notation such as labeling sets with letters, and using the union (∪), and intersection (∩) symbols. Examples: In a class of 90 students, 20 take only Spanish, 15 take only Italian, and 5 take both Spanish and Italian. How many students are NOT enrolled in either Spanish or Italian? Where would this number be placed on the Venn diagram? Solution: Continued on next page

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
·		x = number of students who are not enrolled in either Spanish or Italian
		20 + 5 + 15 + x = 90 40 + x = 90 x = 50 students
		A guidance counselor is planning schedules for 30 students. All students are planning to take one or more languages. Sixteen students say they want to take French, 16 want to take Spanish, and 11 want to take Latin. Five say they want to take both French and Latin. Of these, 3 want to take Spanish as well. Five want only Latin, and 8 want only Spanish.
		How many students want French only? (Solution: 7 students)
		What does the sentence, "All students are planning to take one or more languages." imply regarding the placement of numbers on the Venn diagram? How many students want to take Latin and Spanish? (Solution: 4 students).

Strand 2: Data Analysis, Probability, and Discrete Mathematics Concept 4: Vertex-Edge Graphs

Understand and apply vertex-edge graphs.

In Grade 7, students use vertex-edge graphs to solve real-world problems utilizing Hamilton and Euler paths and circuits.

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to: PO 1. Use vertex-edge graphs and algorithmic thinking to represent and find solutions to practical problems related to Euler/Hamilton paths and circuits. Connections: SS07-S4C1-03, SS07-S4C1-05	M07-S5C2-02. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.	Students should solve the following types of problems: • finding the shortest network connecting specific sites, • finding a minimal route that includes every edge, • finding the shortest route on a vertex-edge graph from one site to another, and • finding the shortest circuit on a vertex-edge graph that makes a tour of specified sites. Example: • Below is a grid graph that represents some streets in Phoenix, AZ. Students are going on a field trip to Phoenix Zoo but district administration will only allow them to go if they can find the shortest route from school to the zoo and prove that they have found the shortest route! (Gas prices are too high these days.) Find the number of ways to get from school to every other location (i.e., vertex) in the unweighted graph, moving only in an east or south direction (i.e., only moving towards the goal location – zoo – from the start location – school). What pattern do you see in these numbers? How many different ways (i.e., paths) are there to get from school to the zoo? Continued on next page

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
		Use the algorithm to find the shortest distance from school to every other vertex in this graph; that is, find the shortest path spanning tree from school. The edge weights represent distance, in miles, of how far one vertex is from another.
		School
		4 8 9 7 11
		8 4 8 8 7
		10 7 6 8 5
		8 9 7 4 Phoenix Zoo
		In that spanning tree, find and identify the shortest path from school to the Phoenix Zoo. Prepare and write an argument for district administration that will prove your findings. Continued on next page

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		
		Solution:
		In the graph below, each number represents the number of ways to get from school to that location, moving only in an eastwardly or southern direction. If you rotate the grid so that the vertex labeled "school" is pointing to the top of your page, you will see the pattern of Pascal's triangle. There are thirty-five different paths to get from school to the zoo.
		School 1 1 1 1
		1 2 3 4 5
		1 3 6 10 15
		1 4 10 20
		35 Phoenix Zoo
		In order to find the shortest path from school to the Phoenix Zoo, students will first discover the shortest path from school to every other vertex in the graph, will systematically label these routes in the graph, creating a shortest path spanning tree, and will identify the desired shortest path route to solve the problem.
		Continued on next page

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		
		 From school there are two adjacent vertices, one to the east and one to the south, that in order to arrive at them, you must travel along the edges emanating from school. The vertex to the east can only be reached one way (see graph above) and the weight on that edge is 7, so seven represents the shortest distance from school to that vertex. As we find the shortest distance from school to every other vertex, that vertex is labeled with a weight (i.e., the shortest distance from school to it) and is highlighted and underlined; at the same time, the edge that is traveled to reach it is dashed (see graph below). This procedure is followed for all vertices along the top edges. There is only one way to reach each of those vertices (moving east). The same holds true for all the vertices down the left most edges of the graph (moving south) from School. There is only one way to reach each of those vertices. In this way, the shortest path spanning tree from school is constructed. Next, students consider what it will take to arrive at the vertex, which is east and then south of school. There are two possible paths (see graph above) leading to that vertex, east-south or south-east; we need to compare them and decide which route is shortest. The shortest route for the east-south path is 7 + 8 = 15. The shortest route for the south-east route is 4 + 5 = 9. Since 9 is the shorter distance, we will want to travel along that route to arrive at this vertex. The tree is extended with a dashed edge to that vertex (along the edge with weight 5) and 9 is highlighted and underlined to indicate that this is the shortest distance from school to this vertex and the path to take is along the dashed route. Continued on next page
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Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
		In the graph below the spanning tree is beginning to take shape.
		School 7 15 19 28
		7 8 9 7 11
		4 9 7 7 7 7 8 7 8 8 7
		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
		22 8 9 7 4
		Phoenix Zoo
		 This process of examining all possible paths and highlighting the shortest one to each vertex is continued until you have determined the shortest path from school to every other vertex in the graph. The path with the shorter weight is added to the existing spanning tree and its corresponding weight is highlighted and underlined.
		Continued on next page

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		
		The graph below shows the shortest path spanning tree from school to the Phoenix Zoo.
		School 7 15 19 28
		4 8 8 9 4 9 11 16 11 11 11 11 11 11 11 11 11 11 11
		4 5 7 7 7 8 7 8 7 17 17 17 17 17 17 17 17 17 17 17 17 1
		10 7 6 8 5
		22 8 9 7 4 34
		Once the shortest path spanning tree from school is constructed, students find the shortest path from school to the Phoenix Zoo by walking backwards in the spanning tree from the zoo. They trace the spanning tree backwards and discover the shortest path.
		Continued on next page

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
		The solid bold edges in the graph below highlight the shortest path from school to the Phoenix Zoo. The total length of this shortest route is 34 miles.
		School 7 15 19 28
		4 7 8 8 9 4 7 9 11 30
		8 4 8 7 8 7 8 7 7 8 7 7 8 7 7 8 7 7 8 7 7 8 7 7 8 7 7 8 7 7 8 7 7 8 7 7 7 8 7 7 7 8 7 7 7 7 7 8 7
		10 7 6 8 5
		22 8 20 9 23 7 30 4 Phoenix Zoo
		Answers will vary. Basically, students should explain that they found the shortest route from school to every other vertex (as shown in the spanning tree) by considering all
		possible ways to arrive at a location and selecting and highlighting the shortest way. Since the zoo was the final destination, they worked backwards from that vertex to find the path, and reported the total distance as 34 miles.
		They should now be allowed to go to the Phoenix Zoo!

Strand 3: Patterns, Algebra, and Functions

Patterns occur everywhere in nature. Algebraic methods are used to explore, model and describe patterns, relationships, and functions involving numbers, shapes, iteration, recursion, and graphs within a variety of real-world problem solving situations. Iteration and recursion are used to model sequential, step-by-step change. Algebra emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically while integrating content from each of the other strands.

In Grade 7, students continue to investigate and analyze patterns and use this information to make conjectures.

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to: PO 1. Recognize, describe, create, and analyze numerical and geometric sequences using tables or graphs; make conjectures about these sequences. Connections: M07-S1C2-01, M07-S3C2-01, M07-S3C3-01	M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language. M07-S5C2-08. Make and test conjectures based on information collected from explorations and experiments.	 Examples: Given the sequence, 2, -1, -4, -7, write a conjecture about the growth of the sequence. Is it growing arithmetically, geometrically, or neither? How do you know? Given the table of values, what relationships can you find? Can you find a rule? 3 1/9 1/27 .

Strand 3: Patterns, Algebra, and Functions Concept 2: Functions and Relationships

Describe and model functions and their relationships.

In Grade 7, students use graphs, tables, and other algebraic techniques to model applied problems with mathematical functions.

Performance Objectives	<u>Process Integration</u>	Explanations and	Example	<u>s</u>	
Students are expected to:					
PO 1. Use a table of values to graph an equation or proportional relationship; describe the graph's characteristics. Connections: M07-S3C1-01, M07-S3C3-	M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.				graph of the function ph is increasing or
04, M07-S3C4-01			X	У	
			-2	-4	-
			-1	-2	-
			0	0	-
			1	2	-
			2	4	-

Strand 3: Patterns, Algebra, and Functions Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations.

In Grade 7, students refine and expand their knowledge of algebraic thinking by solving equations that require more than one step. They translate fluently between graphs, tables, and equations. Students exhibit their understanding of algebra by creating expressions, equations, and inequalities to model a contextual situation.

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		
PO 1. Write a single variable algebraic expression or one-step equation given a contextual situation. Connections: M07-S2C3-02, M07-S3C1-	M07-S5C2-01. Analyze a problem situation to determine the question(s) to be answered.	Meagan spent \$56.58 on three pair of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and helps you determine how much one pair of jeans cost.
O1, M07-S3C3-03, M07-S3C3-05 PO 2. Evaluate an expression containing one or two variables by substituting numbers for the variables. Connections: M07-S1C1-01, M07-S1C2-01, M07-S1C2-05	M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	Values for substitution include whole numbers and integers.
PO 3. Solve multi-step equations using inverse properties with rational numbers. Connections: M07-S1C2-01, M07-S1C3-02, M07-S3C3-01, M07-S3C3-05, M07-S5C2-10	M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Example: • Solve: $\frac{5}{4}$ $n+5=20$ • Solve: 2d+12=5d-3

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		
PO 4. Translate between graphs and tables that represent a linear equation. Connections: M07-S3C2-01	M07-S5C2-02. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.	Does the graph represent the table of values? Explain why or why not.
	M07-S5C2-06. Communicate the	x
	answer(s) to the question(s) in a problem	-2 2
	using appropriate representations, including symbols and informal and formal	-1 1
	mathematical language.	0 0
		1 -1
		2 -2
		<i>y</i> -5 -5 -75

<u>Performance Objectives</u>	<u>Process Integration</u>	Explanations and Examples
Students are expected to: PO 5. Create and solve two-step equations that can be solved using inverse operations with rational numbers. Connections: M07-S1C2-01, M07-S1C3-02, M07-S3C3-01, M07-S3C3-03, M07-S5C2-10	M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Students are expected to use the properties of equality in their solutions. Examples: • Solve the equation: $3a - 2 = 7$ $3a - 2 = 7$ $3a - 2 + 2 = 7 + 2$ Additive Inverse Property $3a = 9$ Identity Property of Addition $\frac{3a}{3} = \frac{9}{3}$ Multiplicative Inverse Property $a = 3$ • Solve: $3x - 8 = 16$
PO 6. Create and solve one-step inequalities with whole numbers. Connections: M07-S1C2-01	M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Contextual problems should be included.

Strand 3: Patterns, Algebra, and Functions Concept 4: Analysis of Change

Analyze how changing the values of one quantity corresponds to change in the values of another quantity.

In Grade 7, students work to recognize how altering a particular quantity will impact a corresponding value. Using graphs and tables allows students to model and visualize change with greater depth of understanding.

Performance Objectives	Process Integration	Explanations and Examples
Performance Objectives Students are expected to: PO 1. Use graphs and tables to model and analyze change. Connections: M07-S1C3-02, M07-S3C2-01, SS07-S4C2-04	Process Integration M07-S5C2-01. Analyze a problem situation to determine the question(s) to be answered.	Example: • Use the graph below to describe the change in y as x increases by 1.
		-5

Strand 4: Geometry and Measurement

Geometry is a natural place for the development of students' reasoning, higher thinking, and justification skills culminating in work with proofs. Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and can be important tools in problem solving. Students use geometric methods, properties and relationships, transformations, and coordinate geometry as a means to recognize, draw, describe, connect, analyze, and measure shapes and representations in the physical world. Measurement is the assignment of a numerical value to an attribute of an object, such as the length of a pencil. At more sophisticated levels, measurement involves assigning a number to a characteristic of a situation, as is done by the consumer price index. A major emphasis in this strand is becoming familiar with the units and processes that are used in measuring attributes.

Concept 1: Geometric Properties

Analyze the attributes and properties of 2- and 3- dimensional figures and develop mathematical arguments about their relationships.

In Grade 7, students expand their investigation of geometric properties to include circles, polygons and three-dimensional shapes and their attributes. Students examine the relationships between and among varying figures.

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		
PO 1. Recognize the relationship between central angles and intercepted arcs; identify arcs and chords of a circle.		Example: • Use circle <i>P</i> to identify an arc and two chords. A B
PO 2. Analyze and determine relationships between angles created by parallel lines cut by a transversal.	M07-S5C2-08. Make and test conjectures based on information collected from explorations and experiments.	Examples of angle relationships that can be explored include but are not limited to: • Alternate interior, alternate exterior, and corresponding angles are congruent. • Same-side (consecutive) interior and same-side (consecutive) exterior angles are supplementary.

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		
PO 3. Draw and classify 3-dimensional figures with appropriate labels showing specified attributes of parallelism, congruence, perpendicularity, and symmetry. Connections: M07-S4C4-04, M07-S4C4-05, M07-S4C4-07	M07-S5C2-01. Analyze a problem situation to determine the question(s) to be answered. M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution. M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations,	Draw a right prism with pentagonal bases
	including symbols and informal and formal mathematical language.	
PO 4. Describe the relationship between the number of sides in a regular polygon and the sum of its interior angles.	M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	An extension of this performance objective could be to explore the pattern of the sum of the exterior angles (360°).
PO 5. Identify corresponding parts of congruent figures.	M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution. M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem	Example: • Given $\triangle ABC \cong \triangle DEF$, list all pairs of congruent angles and sides.
	using appropriate representations, including symbols and informal and formal mathematical language.	A C D F

Strand 4: Geometry and Measurement Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

In Grade 7, students build on their knowledge of translations, reflections, and rotations to construct a combination of two transformations using the coordinate plane.

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
PO 1. Model the result of a double transformation (translations or reflections) of a 2-dimensional figure on a coordinate plane using all four quadrants.	M07-S5C2-04. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.	Students are building on their prior knowledge of graphing figures in the coordinate plane and performing single transformations on figures. Students should apply formal notation using letters to label the vertices of the pre image and A', B', C', A", B", or C" to label the transformed images.
		Given triangle ABC, translate one space to the right. Then reflect it across the x-axis. What are the ordered pairs for triangle A"B"C"?

Strand 4: Geometry and Measurement Concept 3: Coordinate Geometry

Specify and describe spatial relationships using rectangular and other coordinate systems while integrating content from each of the other strands.

In Grade 7, there are no performance objectives in this concept.

Strand 4: Geometry and Measurement

Concept 4: Measurement

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

In Grade 7, students broaden their understanding of area and perimeter to include determining the surface area of solids, recognizing different polygons with the same area or perimeter, and calculating the area and circumference of circles. They select and consider significant information to determine the appropriate degree of accuracy for measurements. The skills and understandings developed at this level prepare students for the next grade level where they are asked to apply their knowledge of area to find the area of composite figures and determine the surface area of common solids.

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to: PO 1. Solve problems involving the circumference and area of a circle by calculating and estimating. Connections: M07-S1C3-01, M07-S1C3-02, M07-S5C2-11	M07-S5C2-11. Use manipulatives and other modeling techniques to defend π as a ratio of circumference to diameter.	Example: • The seventh grade class is building a putting green for a game at the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might you communicate this information to the salesperson to make sure you receive a piece of carpet that is the correct size?
PO 2. Identify polygons having the same perimeter or area.		Example: • Use the figures below to compare the perimeter and area of each shape. 2

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		
PO 3. Calculate the area and perimeter of composite 2-dimensional figures.		Example:
Connections: M07-S1C3-02, M07-S5C1-01		figures are in meters. The curved portion of the second figure is a semi-circle.
		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
PO 4. Determine actual lengths based on scale drawings or maps. Connections: M07-S1C3-02, M07-S1C3-04, M07-S4C1-03, SS07-S4C1-03	M07-S5C2-02. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.	Example: • Julie showed you the scale drawing of her room. If each 2 cm on the scale drawing equals 5 ft, what are the actual dimensions of Julie's room? 5.6 cm 1.2 cm 1.2 cm 2.8 cm

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:		
PO 5. Create a net to calculate the surface area of a given solid. Connections: M07-S1C3-02, M07-S4C1-03, M07-S4C4-07, M07-S5C1-01	M07-S5C2-02. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.	Solids include rectangular and triangular prisms as well as cylinders. Students are expected to use the net to calculate the surface area. Example: • Create the net for a given rectangular prism, and then use the net to calculate the surface area.
PO 6. Identify the appropriate unit of measure to compute the volume of an object and justify reasoning. Connections: M07-S1C3-02	M07-S5C2-03. Identify relevant, missing, and extraneous information related to the solution to a problem.	Students are expected to identify the appropriate unit of measure for volume as well as compute the volume. Example: • A tank at the city park is used to hold the reclaimed water that is used for watering the grass and plants. The tank is 20 feet tall and has a 13 foot diameter. What would be the most appropriate unit of measure to describe the volume or capacity of the tank? Explain why the unit of measure you selected is the most appropriate for the given situation. Compute the volume of the tank.
PO 7. Measure to the appropriate degree of accuracy and justify reasoning. Connections: M07-S4C1-03, M07-S4C4-05, SC07-S1C2-04	M07-S5C2-03. Identify relevant, missing, and extraneous information related to the solution to a problem.	

Strand 5: Structure and Logic

This strand emphasizes the core processes of problem solving. Students draw from the content of the other four strands to devise algorithms and analyze algorithmic thinking. Strand One and Strand Three provide the conceptual and computational basis for these algorithms. Logical reasoning and proof draws its substance from the study of geometry, patterns, and analysis to connect remaining strands. Students use algorithms, algorithmic thinking, and logical reasoning (both inductive and deductive) as they make conjectures and test the validity of arguments and proofs. Concept two develops the core processes as students evaluate situations, select problem solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems.

In Grade 7, students build on their knowledge of how formulas function to develop their own formulas for determining the area of a composite figure. Experience with algorithms will be critical in future math endeavors as students use formulas to solve problems of increased complexity.

Performance Objectives	Process Integration	Explanations and Examples
Students are expected to:		
PO 1. Create an algorithm to determine the area of a given composite figure.	M07-S5C2-08. Make and test conjectures based on information collected from explorations and	Composite figures include figures composed of shapes or parts of shapes put together.
Connections: M07-S4C4-03, M07-S4C4-05	experiments.	Choose one of the figures shown below and write a step by step procedure for determining the area. Find another person that chose the same figure as you did. How are your procedures the same and different? Do they yield the same result?

Strand 5: Structure and Logic Concept 2: Logic, Reasoning, Problem Solving, and Proof

Evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize their applications.

In Grade 7, students analyze problem situations and choose and apply strategies to solve problems. Students are given multiple opportunities to refine their reasoning skills through explaining and defending their solutions.

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:	Some of the Strand 5 Concept 2 performance objectives are listed throughout the grade level document in the Process Integration Column (2nd column). Since these performance objectives are connected to the other content strands, the process integration column is not used in this section next to those performance objectives.	
PO 1. Analyze a problem situation to determine the question(s) to be answered.		
PO 2. Analyze and compare mathematical strategies for efficient problem solving; select and use one or more strategies to solve a problem.		
PO 3. Identify relevant, missing, and extraneous information related to the solution to a problem.		Students are expected to determine what information is needed to solve a problems and if the problem cannot be solved, which information is missing. If possible, students should state their assumption about the missing information and solve the problem using their assumptions.

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:	Some of the Strand 5 Concept 2 performance objectives are listed throughout the grade level document in the Process Integration Column (2nd column). Since these performance objectives are connected to the other content strands, the process integration column is not used in this section next to those performance objectives.	
PO 4. Represent a problem situation using multiple representations, describe the process used to solve the problem, and verify the reasonableness of the solution.		Multiple representations may include but are not limited to numbers, symbols, graphs, equations, pictures, or words.
PO 5. Apply a previously used problem-solving strategy in a new context.		
PO 6. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language. Connections: SC07-S1C4-03, SC07-		Students are expected to use formal notation in expressing algebraic and geometric concepts.
PO 7. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning. Connections: SC07-S1C3-05		Students need multiple opportunities to make inferences, draw conclusions and justify their reasoning using problems from all of the content strands. Students are expected to write justifications and explain their thinking to other students.
PO 8. Make and test conjectures based on information collected from explorations and experiments.		
Connections: SC07-S1C1-01, SC07-S1C2-03, SC07-S1C3-04, SC07-S1C3-06, SC07-S1C3-07		

Performance Objectives	<u>Process Integration</u>	Explanations and Examples
Students are expected to:	Some of the Strand 5 Concept 2 performance objectives are listed throughout the grade level document in the Process Integration Column (2nd column). Since these performance objectives are connected to the other content strands, the process integration column is not used in this section next to those performance objectives.	
PO 9. Solve logic problems using multiple variables and multiple conditional statements using words, pictures, and charts. Connections: M07-S2C3-02	M07-S5C2-06. Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.	Juan, Mary, Bob and Sue live in houses that are blue, red, white, and brown. Their last names are Smith, Garcia, Harris, and Chan. Use clues to match the first and last names and the color of each of their houses.
PO 10. Demonstrate and explain that the process of solving equations is a deductive proof. Connections: M07-S3C3-03, M07-S3C3-05	M07-S5C2-07. Isolate and organize mathematical information taken from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.	Students are expected to state the properties of the real number system to justify each step in a multi-step equation.
PO 11. Use manipulatives and other modeling techniques to defend π (pi) as a ratio of circumference to diameter. Connections: M07-S1C3-01, M07-S4C4-01	M07-S5C2-08. Make and test conjectures based on information collected from explorations and experiments.	Students measure the circumference and diameter of several circular objects in the room (clock, trash can, door knob, wheel, etc.). Students organize their information and discover the relationship between circumference and diameter by noticing the pattern in the ratio of the measures.